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Dynamic assortment in the presence of brand heterogeneity

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ABSTRACT

In this paper, we study the dynamic assortment planning problem in the presence of heterogeneous brands. Over a limited selling season, the retailer sells heterogeneous products from one store brand and one national brand. We use the nested multinomial logit (NMNL) framework to model consumer choice process, in which consumers choose first which brand to buy and then a product within that brand. We formulate this problem using the finite-horizon dynamic programming approach. Using available sales transaction data, we estimate the consumer choice behavior and empirically demonstrate existence of brand heterogeneity. Further, our results suggest that ignoring brand heterogeneity will make the retailer's expected revenues significantly overestimated, and the potential revenue overestimation depends on initial inventories and prices of products of two brands. We finally show that the retailer will benefit from dynamic assortment optimization with the estimated consumer choice model.

1. Introduction

1.1. Motivation and objective

Retailers attach great importance to assortment planning, because the set of products that a retailer carries is one of the main determinants of consumers' store choice and purchasing decisions. To attract more consumers and increase profits, retailers expanded their assortments dramatically in the past decades and have reached unprecedented levels of variety. However, such a high level of variety does not testify to making consumers more satisfied and straightly brings about more operational costs. Thus retailers need to decide a reasonable set of products offering to consumers within a category. In addition, during the selling season, inventories of each product vary because of consumers' choice. Some products are more popular among consumers, which leads these products to run out more quickly before the ending of the selling season. Then the fixed assortment will definitely restrict consumers' purchasing decision, which causes retailers' revenues to decrease. In order to make full use of these limited inventories and increase retailers' revenues, dynamic assortment is introduced. In the setting of dynamic assortment, retailers are able to revise or change assortment selection in each selling period, rationing inventories of some products.

On the other hand, hoping to expand market share, and obtain brand loyalty and bargaining ability, more and more retailers introduce a store brand (or a private label) to all kinds of categories. As Nielsen reported in 2017, the largest markets for store brand products are primarily found in the more mature European retail markets. For example, the store brand's market share in Spain is up to 42% in 2016. In Asia-Pacific area, the store brand's market share is relatively low, but store brand will continue growing up in the future at a higher speed. Many retailers in China, like Vanguard, LianHua and YongHui, are making great efforts for constructing their own store brands. Taking YongHui as an example, in 2018, it formally launched its own store brand whose quantity is about three hundred SKUs, covering kinds of categories, like household items, snack foods, etc. Therefore, there is sufficient evidence to believe that store brands will occupy a significant part of the retail market in the future.

As a result, it is so common that retail categories consist of heterogeneous brands. For example, toothpastes including store brands and national brands are presented in the same shelf space. It is highly possible that consumers treat store brands and national brands unequally because of perceived quality difference. In China, consumers always associate relatively low quality and price with store brands. Conversely, they believe that products of national brands are of high quality. Under this circumstance, the assumption in some models that products in a category are homogeneous is becoming not reasonable. Obviously, the products from different brands are heterogeneous. In the view of consumers, products are divided into different subgroups according to products' brands. Then we could only assume a homogeneous group of products under a brand of a category. We infer that products within one brand are closer substitution to each other than are products from

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another brand. Thus, brand heterogeneity makes the consumer choice process more complex. In this paper, we use a two-stage consumer choice model to depict the consumer choice process.

Without a sharp understanding of consumer choice process, retailers tend to treat store brands and national brands equally in the dynamic assortment planning. However, the reality is different as we have mentioned above. If retailers could not realize brand heterogeneity in theoretical analysis stage and put related results into practice, the actual outcome will certainly deviate from the expectations. This has a negative effect on retailers' operational decisions, which should be examined and avoided in retailers' operation. When reviewing the literature about dynamic assortment, we find that there are still no papers focusing on heterogeneous brands in the context of dynamic assortment. Therefore, we want to fill the gap between literature and reality.

The goal of this paper is to explore the revenue impact of dynamic assortment with heterogeneous brands, which contain a store brand and a national brand. In particular, we want to examine how brand heterogeneity affects the performance of the dynamic assortment. Using sales transaction data from a supermarket chain's data set, we want to estimate consumer choice behavior and demonstrate the potential revenue improvements from dynamic assortment optimization.

1.2. Model and results

We consider a product category with multiple product types from one store brand and one national brand. We characterize consumer choice process as a two-stage nested multinomial logit (NMNL) model (Ben-Akiva and Lerman, 1985). From national brand and store brand, consumers first choose which brand to buy and then select a product type within the chosen brand. Over a finite selling season, the retailer sells limited inventories of products from two brands. To maximize her profit, the retailer needs to make assortment decisions for the whole category. In each selling period, the retailer determines an assortment offered to consumers, which may include products from two brands. We formulate this decision problem as a dynamic assortment planning problem with brand heterogeneity.

Using available sales transaction data from a supermarket chain in China, we first estimate consumers' two-stage choice behavior and preferences. Our estimation results show that consumers have different perceived utilities for products of the store brand and the national brand. In particular, consumers present higher price sensitivity and lower utilities for products of the store brand, in comparison with those of the national brand. Furthermore, our results show that the products within one brand are closer substitution to each other than are products from another brand. That is brand heterogeneity, which means that the retailer should not treat store brand and national brand equally, while characterizing consumer choice process. Otherwise as we have demonstrated, the retailer's expected revenues will be significantly overestimated. We found that the degree of this revenue overestimation is affected by products' initial inventory levels and prices. Such an effect of prices differs between two brands. Our empirical study suggests that the benefits that could be obtained from dynamically optimizing assortment controls to account for choice behavior are significant: the dynamic assortment leads to 11.40% revenue gain in comparison with the offer-all policy, under which all available products are offered to consumers in each period. We further show that the performance of dynamic assortment depends on initial inventory levels and prices of products. Although more inventories could bring out more revenues, we found that there were no significant revenue improvements when the inventory levels exceed some threshold. We finally find that lowering products' prices not only reduces the retailer's expected revenues but also weakens the performance of dynamic assortment. The same price discount on products whose prices are relatively lower in one brand would reinforce the performance of dynamic assortment.

1.3. Contributions

In summary, the main contributions of our paper can be described as follows.

- We first study the dynamic assortment optimization problem with multiple products from one national brand and one store brand.
- A two-stage NMNL model is used to characterize the consumer choice process. We estimate the consumer choice behavior from readily available retailing sales transaction data.
- We empirically demonstrate existence of brand heterogeneity in real retail market.
- Our results suggest that ignoring brand heterogeneity will make the retailer's expected revenues significantly overestimated. The potential revenue overestimation depends on initial inventories and prices of products of national brand and store brand.
- We demonstrate and assess the potential revenue improvements of implementing the dynamic assortment with the estimated consumer choice model.

1.4. Organization of this paper

The remainder of this paper is organized as follows. Section 2 provides a review of the related literature. Section 3 describes the problem and model. Section 4 depicts parameter estimation for the consumer choice model. Section 5 discusses the impact of dynamic assortment with heterogeneous brands by an empirical study. Section 6 concludes this paper.

2. Literature review

Our work is related to three streams of research. The first one includes work on the characteristics of consumers' buying preferences when retailers offer store brand products. The second one is the literature on consumer choice model. The third stream is related to retail assortment planning.

2.1. Consumer purchase preference for store brand

Facing products of store brands and national brands, consumers always have a trade-off between some factors, like quality (Bontems et al., 1999), price (Sethuraman and Cole, 1999), brand loyalty (Anselmsson et al., 2008). There are two main research approaches:

(1) Using a random model to describe consumers' purchase behavior of store brand. Based on a random coefficient Logit model, (Chintagunta et al., 2002) explore effects of introduction of a store brand into a particular product category on both demand side and supply side. Hansen et al. (2006) investigate the behavior of store brand buyers and develop a multi-category brand-choice model which is applied to a set of food and nonfood product categories. They show that there are strong correlations in household preferences for brands across categories.

(2) Using empirical analysis. Baltas (2003) study store brand demand and its determinants using the method of empirical research in a two-stage model to describe the demand for store brand products. Diallo (2012) investigate jointly the effect of store image perceptions, store brand price-image and perceived risk toward store brand on consumers' purchase intention in the context of an emerging market Brazil. Using data from a consumer survey, they find that store image perceptions and store brand price-image influence significantly the purchase intention directly or indirectly via the effect of perceived risk toward store brand. Our study focuses on the operational level and intends to examine the impact of heterogeneous brands on retailers' dynamic assortment.

2.2. Consumer choice process

The multinomial logit (MNL) model has been widely used in demand estimation (e.g., (Vulcano et al., 2012; Newman et al., 2014)) and assortment related issues (Van Ryzin and Mahajan, 1999; Rusmevichientong et al., 2010). These MNL-based models assume a homogeneous group of products in one category-possibly variants of the same product, such as the same garment with different styles, colors, or sizes. The applicability of these models may be limited because most retail categories consist of heterogeneous product subgroups such as coexistence of store brand products and national brand products. And we believe that the products within a subgroup are closer substitution to each other than are products from another subgroup when consumers have strong brand concept. Because of the property of the so-called independence of irrelevant alternatives (IIA) (Anderson and De Palma, 1992), the MNL model fall short of capturing these interactions in a category with heterogeneous product subgroups.

In a category containing heterogeneous product subgroups, a nested multinomial logit (NMNL) model can be a better alternative to depict consumer choice process (Ben-Akiva and Lerman, 1985). Under the NMNL model, customers follow a hierarchical choice process. Consumers first choose among subgroups and then choose a product in the chosen subgroup. The NMNL model provides closed-form choice probabilities much like the MNL model and has been widely used in modeling consumer choice process. Kök and Xu (2011) model brand choice and product type choice using NMNL model with two different hierarchical structures, and study assortment planning and pricing. Wan et al. (2018) build NMNL model to capture consumer choice process under which consumers choose the store at the first level and select the product at the second level. We are interested in a nested structure that reflects brand heterogeneity within a product category.

2.3. Retail assortment planning

Assortment planning is defined by the set of products carried in each store at each point of time. Kök et al. (2008) provide an excellent review of this literature. The assortment planning problem can be divided into static assortment problem and dynamic assortment problem where retailers can revise or change assortment selection as time elapses. Regarding static assortment planning, many decision models have been developed with different demand models. Van Ryzin and Mahajan (1999) derive the optimal assortment policy for a category using the MNL model. Based on this study, Cachon et al. (2005) incorporate consumer search costs into model, and point out that failing to incorporate consumer search into an assortment planning process may cause a retailer to have an assortment with less variety and significantly lower expected profits compared to the optimal solution. Smith and Agrawal (2000) develop a general demand model characterized by the first-choice probabilities and a substitution matrix in an assortment and a methodology for selecting item inventory levels. Chong et al. (2001) present an empirically based modeling framework using an NMNL model to assess the revenue and lost sales implication of alternative category assortments. Mahajan and Van Ryzin (2001) consider assortment planning and inventory management problem under dynamic substitution using a simple utility maximization mechanism. They develop a sample path gradient algorithm to determine the optimal assortment and inventory levels. Gaur and Honhon (2006) use a Hotelling-type location choice model to study assortment planning and inventory management problem in a single category. Kök and Fisher (2007) describe a methodology for demand estimation and substitution rates which are applied to assortment optimization using data from a supermarket chain. All of these papers consider homogeneous product subgroups within categories.

Kök and Xu (2011) study assortment planning and pricing decisions in one category with multiple subgroups of products, which is closer to our formulation. They show whether consumers first choose the product type or first choose the brand under a centralized regime and a decentralized regime, has a critical effect on the optimal management policy. Davis et al. (2014) study a class of assortment optimization problems where customers choose among the offered products according to the nested logit model.

Dynamic assortment planning problem appeared in fashion and apparel retailers at first. Innovative firms such as Zara, Mango, and World Co. created highly responsive and flexible supply chains and cut the design-to-shelf lead time down to 2–5 weeks, which enabled them to make design and assortment selection decisions during the selling season. Caro and Gallien (2007) first study the dynamic assortment optimization. Using the multiarmed bandit for the assortment design problem faced by fast-fashion retailers, they derive bounds on the value function and propose an index-based policy that is shown to be near optimal when there is some prior information on demand. Rusmevichientong et al. (2010) consider an assortment optimization problem subject to a capacity constraint. They propose an adaptive algorithm that learns the unknown parameters from past data and at the same time optimizes the profit. Sauré and Zeevi (2013) study a family of stylized assortment planning problems in which a general random utility model characterizes consumer choice. They develop dynamic policies that balance trade-off between exploration and exploitation and prove that these policies satisfy some performance bounds. Ulu et al. (2012) use a Hotelling locational model with unknown demand distributions that can be discovered by varying the assortment over time. Talebian et al. (2014) propose a stochastic dynamic programming model for simultaneously making assortment and pricing decisions which incorporates demand learning using Bayesian updates. They analytically show that it is profitable for the retailer to use price reductions early in the sales season to accelerate demand learning. All of the dynamic assortment planning papers reviewed above consider learning consumer demand during the selling season.

There are still some papers studying dynamic assortment planning problem without demand learning. Rusmevichientong and Topaloglu (2012) study robust formulations of assortment optimization problems using the MNL choice model under both static and dynamic settings. Caro et al. (2014) propose an attraction model in which product preference weights decay over time. They formulate the assortment packing problem, in which given a collection, a firm must decide in advance the release date of each product to maximize the total profit over the entire selling season. Cınar and Martınez-de Albéniz (2013) model dynamic assortment planning problem by assuming that products lose their attractiveness over time and they have a cost for enhancing the assortment. They characterize the optimal closed-loop policy to maximize firm's profits. Bernstein et al. (2015) propose the idea of assortment customization based on limited inventory conditions and in the presence of heterogeneous customer segments. Note that all these papers do not examine the effect of brand heterogeneity on the dynamic assortment, which is the focus of this paper.

3. Problem statement and model

We study the dynamic assortment optimization problem for one product category (e.g., toothpastes, shampoos and purified water category). Over a finite selling season, the retailer sells a set of heterogeneous products from one store brand and one national brand. Time is discrete and indexed by $t = 1, 2, \ldots, T$. T can be viewed as the number of revision points where the assortment can be refreshed. Let $\mathcal{T} = \{1, 2, \ldots, T\}$. In this paper, we use SB and NB to denote store brand and national brand, respectively. Let N_b be the set of products of brand $b \in B = \{\mathrm{SB}, \mathrm{NB}\}$, which are sold by the retailer. Define $N = N_{\mathrm{SB}} \cup N_{\mathrm{NB}}$. For a given product j, let $\bar{b}(j)$ be the brand of this product. In each period, the retailer determines an assortment of products from two brands to offer to consumers, given available inventory. There is no replenishment during the selling season. For each period t, let S_{bt} denote the assortment of products of brand $b \in B$. Hereafter, we define $S_t = \cup_{b \in B} S_{bt}, \forall t \in \mathcal{T}$. The retailer needs to determine an optimal assortment in each period, so that the expected revenues are maximized over the selling season.

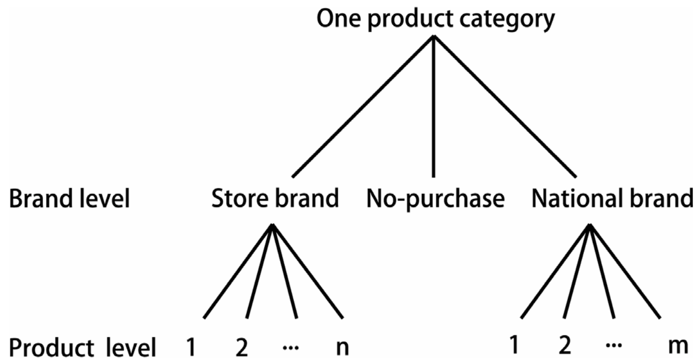

Fig. 1. Two-stage consumer choice model.

3.1. Consumer choice model

Given assortment S_t, consumers make purchase decisions following a two-stage hierarchical choice process. In this paper, we use the nested multinomial logit (NMNL) model to depict consumer choice behavior (see Fig. 1 for illustration). In this model, a consumer first decides which brand or the no-purchase option to choose. If the consumer chooses a brand instead of no-purchase option, then he decides which product offered by this brand to purchase.

We assume consumers are utility maximizers and individual consumer's utilities for alternatives are random variables. Let U_{it} be the utility of a consumer purchasing product i in period t, which can be decomposed into two parts: a representative v_{it} and a random component ϵ_{it}. That is,

$$U_{it} = v_{it} + \epsilon_{it}.$$

The representative component v_{it} is deterministic, which is modeled as a linear-in-parameter combination of observable attributes:

$$v_{it} = \beta_{0i} + \beta_{1i} p_{it} + \beta_{2i} f_i, \tag{1}$$

where β_{0i} is a specific constant associated with product i; β_{1i} and β_{2i} are unknown parameters. These parameters will be estimated from retailers' sales transaction data. Let $\boldsymbol{\beta}$ be parameter vector in the utility function. p_{it} is the price of product i in period t; f_i is one specific feature of product i.

In each period, the utility of no-purchase is defined as

$$U_{0t} = v_{0t} + \epsilon_{0t}.$$

In the NMNL model, it is assumed that the random components ϵ_{it}s and ϵ_{0t}s are independent and identically distributed random variables with a Gumbel (or double-exponential) distribution (Gumbel, 1958). The cumulative density function of the Gumbel distribution is $F(x) = \exp[-\exp(x/\mu_1 + \gamma)]$, where γ is the Euler's constant (=0.5722...) and μ_1 is the scale parameter. Here μ_1 is a positive constant, with a higher value of μ_1 corresponding to a higher degree of heterogeneity among the population. We let $\mu_1 = 1$ to simplify exposition. As we know, the Gmubel distribution has some useful analytical properties, the most important of which is that the distribution of the maximum of n independent Gumbel random variables with the same scale parameter $\mu_1 = 1$ is also a Gumbel random variable. The assumption of the Gumbel distribution in the NMNL model, while restrictive, leads to a simple form of the choice probabilities. Without loss of generality, we assume that no-purchase utility is zero; i.e., $v_{0t} = 0$.

Under the NMNL, the probability that a consumer chooses product i offered by brand b in period t, is given by

$$P^t_{ib}(\boldsymbol{\beta}, \mu, S_t) = P_{bt}(\boldsymbol{\beta}, \mu, S_t) \cdot P_t(i|b, \boldsymbol{\beta}, S_{bt}), \forall i \in S_{bt}, \forall b \in B, \forall t \in \mathcal{T},$$

where $P_{bt}(\boldsymbol{\beta}, \mu, S_t)$ denotes the probability that a consumer chooses brand b in period t, which depends on parameters β, μ, and assortment S_t. It is given by

$$P_{bt}(\boldsymbol{\beta}, \mu, S_t) = \frac{[\sum_{j \in S_{bt}} \exp(v_{jt})]^{1/\mu}}{\sum_{b' \in B}[\sum_{k \in S_{b't}} \exp(v_{kt})]^{1/\mu} + 1},$$

where μ is the scale parameter that controls the brand heterogeneity. $P_t(i|b, \boldsymbol{\beta}, S_{bt})$ is the probability of a consumer choosing product i in an assortment S_{bt} of brand b. It can be defined using the MNL model,

$$P_t(i|b, \boldsymbol{\beta}, S_{bt}) = \frac{\exp(v_{it})}{\sum_{k \in S_{bt}} \exp(v_{kt})}.$$

We let $P^t_0(\boldsymbol{\beta}, \mu, S_t)$ be the probability that a consumer does not purchase from the offered assortment; i.e.,

$$P^t_0(\boldsymbol{\beta}, \mu, S_t) = \frac{1}{\sum_{b' \in B}[\sum_{k \in S_{b't}} \exp(v_{kt})]^{1/\mu} + 1}.$$

Note that if $\mu = 1$, the NMNL model reduces to the standard MNL model, where all products of different brands form a homogeneous set. The value of μ will be also estimated from retailer's sales transaction data.

3.2. The dynamic assortment optimization model

In this section, we give the dynamic programming formulation for the dynamic assortment optimization problem. Given a limited initial inventories of products of two brands, the retailer needs to determine the assortments for both SB and NB in each selling period, to maximize the expected revenues over the whole selling season. Without loss of generality, we assume that the salvage value for unsold units at the end of the selling season is zero (it is a common assumption in the literature).

We assume that the number of consumer arrivals in each period t follows a Poisson distribution with mean λ_t (arrival rate). In this paper, we consider an identical arrival rate λ over all the T periods. We assume a sufficiently small time interval such that in each period there is at most one consumer arrival (this assumption is the same as in Talluri and van Ryzin (2004)). Thus, the probability of no consumer arrival is $1 - \lambda$. We further assume that a product's prices remains unchanged over time; that is, we do not consider pricing decision in the dynamic assortment.

For each product i, let y_{i0} and y_{it} respectively be the initial inventory levels at the beginning of the selling season and in period t. Let $\boldsymbol{y_t}$ be the corresponding inventory vector in period t, $\boldsymbol{y_t} = \{y_{it} : i \in N\}$. Given inventory $\boldsymbol{y_t}$, let $\overline{S}_b(\boldsymbol{y_t})$ be the set of possible assortments of brand b in period t. Define the value function $\Pi_t(\boldsymbol{y_t})$ as the maximum revenue obtainable from periods $t, t{+}1, \ldots, T$, given that for each product i, there are y_{it} inventory units remaining at time t. Then, the Bellman equation for $\Pi_t(\boldsymbol{y_t})$ is

$$\Pi_t(\boldsymbol{y_t}) = \max_{S_{bt} \subset \overline{S}_b(\boldsymbol{y_t})} \Bigg\{ \sum_{b \in B} \sum_{i \in S_{bt}} \lambda P^t_{ib}(\boldsymbol{\beta}, \mu, S_t)[p_i + \Pi_{t+1}(\boldsymbol{y_t} - \boldsymbol{e_i})]$$

$$+ (\lambda P_0^t(\boldsymbol{\beta}, \mu, S_t) + 1 - \lambda)\Pi_{t+1}(\mathbf{y}_t)\Big\},$$

and the boundary condition is

$$\Pi_{T+1}(\mathbf{y}_{T+1}) = 0.$$

The first term is the expected value from an arriving consumer (an arrival occurs with probability λ). For a given assortment S_t, this term accounts for the probability of selling one unit of product i, earning a revenue of p_i from the sale, plus the revenue-to-go function in period $t+1$ evaluated at the current inventory level minus the unit sold in period t. e_i is a vector with the ith component being 1 and others being 0s. The second term accounts for the possibility that no consumer arrives (with probability $1-\lambda$), or the arriving consumer does not make a purchase. In this case, the revenue is the revenue-to-go function in period $t+1$ evaluated at the current vector of inventory levels.

4. Parameter estimation

In this section, we discuss how to estimate parameters in the consumer choice model, using available sales transaction data. Let z_{it} be the number of purchases of product i observed in period t. We denote the total number of observed purchases in period t by m_t; i.e., $m_t = \sum_{i\in N} z_{it}$. We denote vector of parameters $\boldsymbol{\beta}$, μ and λ by $\boldsymbol{\phi}$; i.e., $\boldsymbol{\phi} = (\boldsymbol{\beta}, \mu, \lambda)$.

A major incompleteness in the sales transaction data is that there is no information about the number of consumers who arrive in a period but do not make a purchase; that is, the total number of transactions in a given period is a censored approximation to the true demand of the period. We treat the no-purchase option as a separate product that is always available. Following Abdallah and Vulcano (2016) and Newman et al. (2014), we can form the incomplete data likelihood function, $L(\boldsymbol{\phi})$ as:

$$L(\boldsymbol{\phi}) = \prod_{t=1}^{T}\Bigg(P(m_t \text{ customers buy in period } t|\ \boldsymbol{\phi}) \times \frac{m_t!}{\prod_{i\in N} z_{it}!}\prod_{j\in S_t}\left[\frac{P^t_{j\bar{b}(j)}(\boldsymbol{\beta}, \mu, S_t)}{\sum_{i\in S_t} P^t_{i\bar{b}(i)}(\boldsymbol{\beta}, \mu, S_t)}\right]^{z_{jt}}\Bigg),$$

where

$$P(m_t \text{ customers buy in period } t|\ \boldsymbol{\phi}) = \frac{[\lambda \sum_{i\in S_t} P^t_{i\bar{b}(i)}(\boldsymbol{\beta}, \mu, S_t)]^{m_t}\exp[-\lambda \sum_{i\in S_t} P^t_{i\bar{b}(i)}(\boldsymbol{\beta}, \mu, S_t)]}{m_t!}.$$

Taking logarithm, we can write the log-likelihood function $LL(\boldsymbol{\phi})$ as

$$\begin{aligned}
LL(\boldsymbol{\phi}) &= \sum_{t=1}^{T}\log\Bigg(\frac{[\lambda \sum_{i\in S_t} P^t_{i\bar{b}(i)}(\boldsymbol{\beta}, \mu, S_t)]^{m_t}\exp[-\lambda \sum_{i\in S_t} P^t_{i\bar{b}(i)}(\boldsymbol{\beta}, \mu, S_t)]}{\prod_{i\in N} z_{it}!} \prod_{j\in S_t}\left[\frac{P^t_{j\bar{b}(j)}(\boldsymbol{\beta}, \mu, S_t)}{\sum_{i\in S_t} P^t_{i\bar{b}(i)}(\boldsymbol{\beta}, \mu, S_t)}\right]^{z_{jt}}\Bigg)\\
&= \sum_{t=1}^{T}\Bigg[\log\Bigg(\frac{[\lambda \sum_{i\in S_t} P^t_{i\bar{b}(i)}(\boldsymbol{\beta}, \mu, S_t)]^{m_t}\exp[-\lambda \sum_{i\in S_t} P^t_{i\bar{b}(i)}(\boldsymbol{\beta}, \mu, S_t)]}{\prod_{i\in N} z_{it}!}\Bigg) + \sum_{j\in S_t}\log\left[\frac{P^t_{j\bar{b}(j)}(\boldsymbol{\beta}, \mu, S_t)}{\sum_{i\in S_t} P^t_{i\bar{b}(i)}(\boldsymbol{\beta}, \mu, S_t)}\right]^{z_{jt}}\Bigg]\\
&= \sum_{t=1}^{T}\Bigg[m_t\log\lambda + m_t\log\Bigg(\sum_{i\in S_t} P^t_{i\bar{b}(i)}(\boldsymbol{\beta}, \mu, S_t)\Bigg) - \lambda\sum_{i\in S_t} P^t_{i\bar{b}(i)}(\boldsymbol{\beta}, \mu, S_t)\\
&\quad - \sum_{i\in N}\log(z_{it}!) + \sum_{j\in S_t} z_{jt}\log\Big(P^t_{j\bar{b}(j)}(\boldsymbol{\beta}, \mu, S_t)\Big) - \sum_{j\in S_t} z_{jt}\log\Bigg(\sum_{i\in S_t} P^t_{i\bar{b}(i)}(\boldsymbol{\beta}, \mu, S_t)\Bigg)\Bigg]\\
&= \sum_{t=1}^{T}\Bigg[m_t\log\lambda - \lambda\sum_{i\in S_t} P^t_{i\bar{b}(i)}(\boldsymbol{\beta}, \mu, S_t) - \sum_{i\in N}\log(z_{it}!) + \sum_{j\in S_t} z_{jt}\log\Big(P^t_{j\bar{b}(j)}(\boldsymbol{\beta}, \mu, S_t)\Big)\Bigg].
\end{aligned}$$

The last equality holds since $m_t = \sum_{j\in N} z_{jt}$, and thus $m_t\log\Big(\sum_{i\in S_t} P^t_{i\bar{b}(i)}(\boldsymbol{\beta}, \mu, S_t)\Big) = \sum_{j\in N} z_{jt}\log\Big(\sum_{i\in S_t} P^t_{i\bar{b}(i)}(\boldsymbol{\beta}, \mu, S_t)\Big)$.

To estimate parameter $\boldsymbol{\beta}$, μ and λ, we use the Markov Chain Monte Carlo (MCMC) algorithm, where a Bayesian method is utilized for sampling (Musalem et al., 2009, 2010). The MCMC estimation approach is described in Algorithm 1. Let $\pi(\boldsymbol{\phi})$ be the prior distribution of $\boldsymbol{\phi}$. A uniform prior distribution is assumed for each parameter. In addition, we use the prior distribution as the proposal distribution.

Algorithm 1. The MCMC Estimation Method.

Input sales transaction data D.
Set the prior distribution $\pi(\boldsymbol{\phi})$ of $\boldsymbol{\phi}$.
Sample $\boldsymbol{\phi}^{(0)}$ according to $\pi(\boldsymbol{\phi})$.
Set $r = 1$.
while $(r \le 100,000)$
  Set $\boldsymbol{\phi}^{(r)} = \boldsymbol{\phi}^{(r-1)}$.
  for each parameter $\boldsymbol{\phi}_j$, **do**
    Sample $\boldsymbol{\phi}_j^*$ from the proposal distribution.
    Generate $\boldsymbol{\phi}_j^{*(r)}$.
    Generate a random number ω from interval $[0, 1]$.
    if $\omega < \mathrm{P}\{\text{accept } \boldsymbol{\phi}_j^{*(r)}\} = \min\{\exp(LL(\boldsymbol{\phi}_j^{*(r)}) - LL(\boldsymbol{\phi}^{(r)})), 1\}$
      Update $\boldsymbol{\phi}^{(r)}$ using $\boldsymbol{\phi}_j^{*(r)}$.
    end if
  end for
  Set $r = r + 1$.
end while
Set $\hat{\boldsymbol{\phi}} = \frac{1}{10,000}\sum_{t=90,001}^{100,000}\boldsymbol{\phi}^{(r)}$.

In our implementation, we choose the starting point for each parameter by simply taking a random point from a specific uniform distribution. To sample parameter $\boldsymbol{\phi}$, we use a hybrid Gibbs sampling method, which embeds the Metropolis–Hastings algorithm within Gibbs sampling. Gibbs sampling breaks down the problem by drawing samples for each parameter directly from that parameter's conditional posterior distribution, or the probability distribution of a parameter given a specific value of other parameters. However, sampling $\boldsymbol{\phi}$ from their conditional posterior distributions is challenging. We use the independent Metropolis–Hastings algorithm to sample $\boldsymbol{\phi}$.

Particularly, at iteration r, Gibbs sampling consists of a series of 26 (the number of parameters) steps, with step j of iteration r corresponding to an update of the subvector $\boldsymbol{\phi}_j$ conditional on given all the other elements of $\boldsymbol{\phi}$. Let $\boldsymbol{\phi}^{(r)}$ be the parameter value sampled at iteration r, which is initialized equal to $\boldsymbol{\phi}^{(r-1)}$. Suppose we are sampling the jth parameter (say $\boldsymbol{\phi}_j$) in $\boldsymbol{\phi}$. As a result, let ϕ_j^* denote the sampled value of this parameter. At iteration r, we let $\boldsymbol{\phi}_j^{*(r)}$ denote the updated parameter vector after the jth parameter is sampled; i.e., the jth component is replaced by ϕ_j^*. In the Metropolis–Hastings algorithm, $\boldsymbol{\phi}_j^{*(r)}$ is accepted with probability

$$\mathrm{P}\{\text{accept } \boldsymbol{\phi}_j^{*(r)}\} = \min\{\exp(LL(\boldsymbol{\phi}_j^{*(r)}) - LL(\boldsymbol{\phi}^{(r)})), 1\}.$$

We set $\boldsymbol{\phi}^{(r)}$ equal to $\boldsymbol{\phi}_j^{*(r)}$ if we accept $\boldsymbol{\phi}_j^{*(r)}$. Otherwise, we keep $\boldsymbol{\phi}^{(r)}$ unchanged.

The MCMC estimation procedure terminates with 100,000 iterations. We discard the first 90,000 estimation results. The average of the last 10,000 sampled $\boldsymbol{\phi}^{(r)}$ is used as the estimand $\hat{\boldsymbol{\phi}}$; i.e., $\hat{\boldsymbol{\phi}} = \frac{1}{10,000}\sum_{t=90,001}^{100,000}\boldsymbol{\phi}^{(r)}$.

Table 1
Data for estimation.

Item	Store brand: JiaHui				National brand: YiBao			
	P1	P2	P3	P4	P5	P6	P7	P8
Volume	350 mL	550 mL	4 L	5 L	350 mL	555 mL	1.555 L	4.5 L
Price (RMB)	1.6	2	5.9	6.5	1.3	1.5	3.1	7.8
# of transactions	390	583	590	1112	356	3328	1381	1129

5. Empirical analysis

In this section, we first report the estimation results on a real-world sales transaction data set from a large supermarket chain in Shanghai, China. In particular, we want to estimate consumer choice behavior and examine how the brand heterogeneity affects the performance of dynamic assortment. We then assess the potential revenue improvements from dynamic assortment. To measure the performance of dynamic assortment, we calculate the percentage of revenue improvements relative to a benchmark called offer-all policy, under which all available products are offered to consumers in each period.

5.1. Estimation results

In order to estimate parameters in the consumer choice model, we obtained sales transaction data from one of the largest supermarket chains for consumer products in Shanghai, China. We were provided data with one SB (called “JiaHui") and one NB (called “YiBao") from the purified water category. These two brands are both top brands in this supermarket. Under these brands, there are diversified products with different characteristics. For each brand, we selected four products that differ in volumes. These eight products are labeled as $\mathrm{P}_i, \forall i = 1, 2, \ldots, 8$. The data was collected from a store in downtown Shanghai, and covered 122-day sales transaction, ranging from June 1 to September 30, 2015. All these eight products of two brands were sold during this period. The data description is shown in Table 1. In this empirical study, we include prices and volume in definition of consumer utility. The last row of Table 1 reports the number of transactions of each product during the considered selling season.

For each transaction, the sales data includes information of sales time, selling price, and the number of units sold. The sales transaction data can give us the assortments S_t every day. The assortments S_t can vary in the considered selling season of 122 days. But we assume that the assortments S_t do not vary during different periods of one day, because the retailer did not record this information. That is, the assortments in one day remain unchanged. Note that our proposed model and methodology do not require that the assortments cannot change in one day. As previously stated, we assume that each consumer purchases at most one unit product in each period. Time slice is determined so as to guarantee that at most one consumer arrives in each period. For eight products of two brands, we recorded time span between two adjacent transactions and conducted the statistical analysis. We found that it is small enough to set time slice equal to 30 s, so that at most one purchase occurs in this time slice. The supermarket is operated from 7:30 a.m. to 10:00 p.m. every day. Thus, the selling season is divided into $T = 122 * 14.5 * 3600/30 = 212,280$ time periods.

After preprocessing data, we finally applied the MCMC algorithm to estimate the parameters. We assume a uniform prior distribution for each parameter, which is detailed in Table 2. The estimation results are shown in Table 3.

From these results, we find that there are significant distinctions between SB products and NB products. The price sensitivity of the SB products is higher than that of the NB products. This makes sense. In Chinese retail market, the popularity of SB is far behind that in mature European retail markets. However, nowadays more and more retailers launch their own store brands, such as “Better Living" of LianHua, “Family of Run" of Vanguard, etc. A majority of consumers attach low price and low quality to SB. Therefore, it is reasonable that consumers are more sensitive to prices of SB products.

It is interesting to interpret the relative values of the coefficients as indicators of the sensitivity of the choices. Let us look at P1 and P5. These two products have the same volume, 350 mL. The mean utilities of these two products are, respectively,

$$v_{1t} = 3.2062 - 5.1192 p_{1t} + 0.7727 \times 0.35 = 3.4766 - 5.1192 p_{1t} \quad \text{and}$$
$$v_{5t} = -0.2783 - 1.1757 p_{5t} + 0.8334 \times 0.35 = -0.0116 - 1.1757 p_{5t}.$$

We then have $v_{1t} > v_{5t}$, when $p_{1t} < 0.6814 + 0.2297 p_{5t}$. The current price of P5 is 1.3RMB. Therefore, setting p_{1t} less than 0.98RMB is able to make P1 a more attractive alternative (on average). However, the current selling price of P1 is 1.6RMB. This type of analysis could be useful for the retailer to determine price strategies of SBs.

Further we can see the volume coefficients have the positive sign. That is, the retailer is able to increase her revenues through product volume increment. More importantly, the factor of “volume" has a larger effect on the products of SB. If such a volume-based strategy is implemented, the retailer may focus on those products with larger $\hat{\beta}_2$, e.g., products P2 and P3.

Substituting parameters' values into Eq. (1), we can get the mean utility of each product. For instance, the mean utility of P1 is $v_1 = 3.2062 - 5.1192 \times 1.6 + 0.7727 \times 0.35 = -4.7140$. The mean utility of each product is shown in the last row of Table 3. The results show that products of SB have lower utilities than those of NB. Therefore, the SB products have smaller probabilities of being chosen by consumers.

Finally we can see that the estimated value of μ is greater than 1. It is concluded that products across two brands are less replaceable than products within a brand. Note that under the MNL model, the products of SB and NB are put in one product pool and treated equally. Naturally, we conjecture that brand heterogeneity may affect the performance of the dynamic assortment. Such an effect will be examined explicitly in the next section.

5.2. Assessing effect of brand heterogeneity on revenue improvements

The above estimation results have demonstrated that consumers have different utilities for SB and NB products. If such a hierarchical choice behavior is ignored, the expected revenues from the assortment planning may deviate from the reality. We next used the estimation results fit from the real-word sales transaction data to assess the potential revenue overestimation from brand heterogeneity. Toward this end, we respectively solved the dynamic assortment optimization problem with the MNL and NMNL models. To solve the dynamic assortment optimization problem, we need to set the initial inventory level of each product at the beginning of the selling season. To this end, we assume all products are offered to consumers in each period and set its initial inventory equal to the rounded expected sales quantity of each product, i.e.,

$$y_{i0} = \lceil \hat{\lambda} \cdot P_{ib} \cdot T \rceil.$$

where P_{ib} is the probability of choosing product i of brand b, given that all the products are displayed to the customers. $\hat{\lambda}$ is the one determined in Section 5.1. Now the setting of the initial inventory depends on the length of the selling season T. A larger T will give a larger initial inventory. Thus, a larger T will produce a large-scale dynamic programming, which is required for solving the dynamic assortment

Table 2
Uniform prior distributions.

Parameter	Store brand: JiaHui				National brand: YiBao			
	P1	P2	P3	P4	P5	P6	P7	P8
β_0	(3, 4)	(−2, −1)	(−1, 0)	(2, 3)	(−1, 0)	(0, 1)	(−2, −1)	(1, 2)
β_1	(−6, −5)	(−2, −1)	(−2, −1)	(−2, −1)	(−2, −1)	(−0.1, 0)	(−0.1, 0)	(−1, 0)
β_2	(0.5, 1.5)	(2, 3)	(2, 3)	(0.5, 1.5)	(0.5, 1.5)	(0, 1)	(1, 2)	(0, 1)
μ	(4, 5)							
λ	(0, 0.1)							

Table 3
Estimated parameters.

Item	Store brand: JiaHui				National brand: YiBao			
	P1	P2	P3	P4	P5	P6	P7	P8
$\hat{\beta}_0$	3.2062	−1.8639	−0.9194	2.8945	−0.2783	0.8822	−1.3001	1.3577
$\hat{\beta}_1$	−5.1192	−1.6170	−1.9444	−1.8845	−1.1757	−0.0143	−0.0170	−0.5888
$\hat{\beta}_2$	0.7727	2.6304	2.5332	1.4797	0.8334	0.6168	1.0774	0.7229
$\hat{\mu}$	4.6069							
$\hat{\lambda}$	0.0348							
$\hat{v}$	−4.7140	−3.6512	−2.2585	−1.9557	−1.5150	1.2031	0.3226	0.0180

Table 4
Expected revenues under two consumer choice models.

Choice model	Expected revenues		Revenue improvements
	Offer-all	Dynamic assortment	
MNL	56.67	63.58	12.21%
NMNL	52.09	58.03	11.40%
Revenue overestimation	8.79%	9.58%	

problem. To obtain a tradeoff between the required computing effort and reporting meaningful managerial insights from the dynamic assortment, we set $T = 600$ in our experiment. As a result, the initial inventory is $\mathbf{y}_0 = (1, 1, 2, 3, 1, 6, 3, 2)$. The price of each product is set to the real one derived from the sales transaction data; i.e., the price vector $\mathbf{p}_0 = (1.6, 2, 5.9, 6.5, 1.3, 1.5, 3.1, 7.8)$. As a comparison benchmark, we also solved an assortment optimization problem under the offer-all policy, where all the available products are offered to consumers.

Table 4 reports the expected revenues from the assortment optimization under two consumer choice models. In the last row of Table 4, we show the percentage of revenue overestimation when the retailer treats SB and NB equally.

The results in Table 4 clearly show that the dynamic assortment with the MNL model yields larger expected revenues than that with the NMNL model. In other words, the expected revenues will be significantly overestimated if consumers' hierarchical choice behavior is ignored. Under offer-all and dynamic assortment strategies, the expected revenues are overestimated 8.79% and 9.58%, respectively. The results demonstrate that it is necessary for the retailer to take into consideration the impact of brand heterogeneity, while capturing consumer choice process. More importantly, we can see that the retailer does benefit from dynamically adjusting products offered to consumers. In comparison with the offer-all policy, the dynamic assortment leads to 11.40% increase of the expected revenues, when the estimated NMNL model is used.

The above results have demonstrated that customers' hierarchical choice behavior incurs revenue overestimation. We further examined whether the degree of revenue deviation is related to the initial inventory level. To this end, we solved the assortment optimization problem with different levels of initial inventory under dynamic assortment. In particular, we want to show how the revenue overestimation evolves with change of inventories of SB and NB, respectively. We denote $\mathbf{y}_0 = (1, 1, 2, 3, 1, 6, 3, 2)$ in the last comparison study as "basic inventory". Fig. 2 depicts the results. In Fig. 2(a), the inventory levels of SB products are simultaneously increased, or decreased based on the basic inventory level $\mathbf{y}_0$. Fig. 2(b) applies to NB products. Results in Fig. 2(a) show that the percentage of revenue overestimation rises up, when we simultaneously increase the inventory levels for SB products. The same observation can be derived from the case of adjusting inventory levels of NB products (see Fig. 2(b)).

We further examine whether simultaneously adjusting prices of SB, or NB products will affect the revenue overestimation under dynamic assortment. To this end, we adjusted basic price vector $\mathbf{p}_0$ by a discount factor δ to create six new price scenarios. The results are reported in Fig. 3. "SB-20%" means that the discount factor $\delta = 20\%$ applies to all the SB products. That is, $\mathbf{p}'$={1.6×(1–0.2), 2×(1–0.2), 5.9×(1–0.2), 6.5×(1–0.2), 1.3, 1.5, 3.1, 7.8} = (1.28, 1.6, 4.72, 5.2, 1.3, 1.5, 3.1, 7.8). Results in Fig. 3 show that under these price scenarios, the expected revenues are all overestimated, when the retailer ignore customers' perceived utility difference between SB and NB products. We can see that this revenue overestimation evolves differently for SB and NB. The percentage of revenue overestimation will decline with increasing price discount factor on SB products. Conversely, increasing price discount factor of NB products would enlarge the percentage of revenue overestimation. Furthermore, we find that the same price discount factor on NB products will produce greater revenue overestimation than on SB products.

5.3. Impact of initial inventory on the dynamic assortment performance

In this section, we assess the impact of initial inventory on revenue improvements from the dynamic assortment. In particular, we aim to examine the following two issues:

- How do revenue improvements from the dynamic assortment change, when we simultaneously adjust initial inventory levels of SB and NB products?
- Suppose the storage space is fixed. If we increase inventory for products of one brand and decrease inventory for another brand, how will the revenue improvements evolve?

To examine these above issues, we solved the dynamic assortment problem under seven scenarios of initial inventory:

- Scenario 2. It corresponds to the basic inventory $\mathbf{y}_0$.
- Scenarios 1 and 3. They are created by increasing or decreasing one unit inventory for all the products, respectively.
- Scenarios 4 and 5. Based on the basic inventory, we add one unit inventory to SB products and decrease by one unit inventory for NB products in scenario 4. Conversely, in scenario 5, inventories of SB and NB products are respectively decreased and increased one unit.

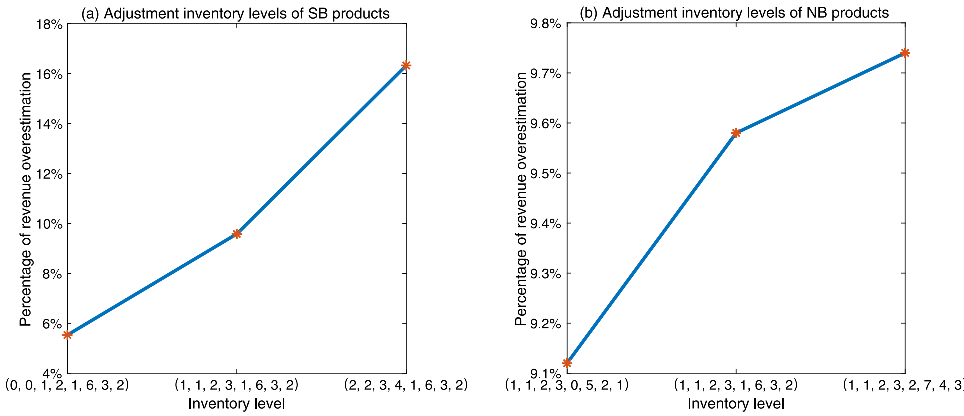

Fig. 2. Effect of inventory levels on revenue overestimation.

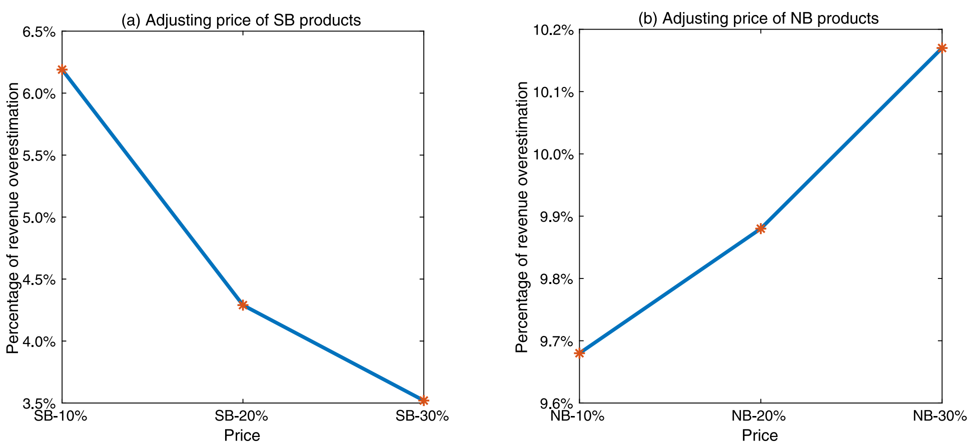

Fig. 3. Effect of Price on revenue overestimation.

Table 5
Impact of initial inventory on dynamic assortment.

Scenario	Initial inventory	Expected revenues		Revenue improvements
		Offer-all	Dynamic assortment	
1	(2, 2, 3, 4, 2, 7, 4, 3)	55.33	69.16	24.99%
2	(1, 1, 2, 3, 1, 6, 3, 2)	52.09	58.03	11.40%
3	(0, 0, 1, 2, 0, 5, 2, 1)	38.52	39.00	1.25%
4	(2, 2, 3, 4, 0, 5, 2, 1)	51.02	55.49	8.77%
5	(0, 0, 1, 2, 2, 7, 4, 3)	49.11	57.16	16.39%
6	(4, 1, 2, 3, 1, 6, 3, 2)	52.13	58.14	11.54%
7	(8, 1, 2, 3, 1, 6, 3, 2)	52.13	58.15	11.54%

- Scenarios 6 and 7. They are created by adding three and seven units inventory to product 1.

As a comparison benchmark, we also solved the assortment optimization problem under the offer-all policy. Table 5 summarizes the results.

We first answer question 1 (refer to results of scenarios 1–3). In comparison with the offer-all policy, the dynamic assortment yields more revenue improvements, when we increase initial inventory levels of all the products of two brands. This can be explained as follows. In scenario 1, the initial inventory is larger than the expected demand in the selling season. The retailer has more flexibility in implementation of dynamic assortment. Therefore, there is larger revenue increase after rationing inventory. However, when the inventory level of any product is greater than some threshold, there is no significant improvement in expected revenues because of the limited demand. This circumstance is shown in scenarios 6 and 7. When the initial inventory of product 1 increases from 4 units to 8 units, the expected revenues under two policies increase a little. Therefore, increasing initial inventory could bring out more revenues, but its function is limited. In scenario 3, the initial inventory is very small and thus the retailer has enough time to sell out these products. All these products are likely to be included in the optimal assortment. As a result, the dynamic assortment is similar to the offer-all policy, and the percentage of revenue improvements will become very small.

We next answer question 2 (refer to results of scenarios 4 and 5). As we can see, the dynamic assortment yields a larger percentage of revenue improvements in scenario 5 than in scenario 4. Because higher utilities are associated with NB products, they are more possible to be chosen by consumers. That is, given a fixed storage space, the retailer prefers to add inventory for NB products in order to maximize the expected revenues.

Suppose the retailer has a chance of storing extra one unit inventory in the warehouse. Then which product should the retailer choose? For this purpose, we further examine the effect of extra one unit inventory on expected revenues. Table 6 reports the percentage of revenue improvements, which are calculated based on the expected revenues with basic inventory $\mathbf{y}_0 = (1, 1, 2, 3, 1, 6, 3, 2)$. In Table 6, we consider eight scenarios, where each one corresponds to adding one unit inventory to only one product. Scenarios 1–4 apply to SB products, whereas scenarios 5–8 are for NB products.

As one may observe, increasing one extra unit inventory to these eight products does yield revenue improvements, under the dynamic assortment. More inventory makes the retailer have more flexibility to implement the dynamic assortment. Then the expected revenues

Table 6
Impact of increasing one unit inventory on revenue improvements.

Scenario	Initial inventory	Percentage of revenue improvements	
		Offer-all	Dynamic assortment
1	(2, 1, 2, 3, 1, 6, 3, 2)	0.03%	0.13%
2	(1, 2, 2, 3, 1, 6, 3, 2)	−0.12%	0.30%
3	(1, 1, 3, 3, 1, 6, 3, 2)	2.51%	4.12%
4	(1, 1, 2, 4, 1, 6, 3, 2)	2.97%	4.89%
5	(1, 1, 2, 3, 2, 6, 3, 2)	−0.14%	0.10%
6	(1, 1, 2, 3, 1, 7, 3, 2)	−0.81%	0.17%
7	(1, 1, 2, 3, 1, 6, 4, 2)	0.65%	2.26%
8	(1, 1, 2, 3, 1, 6, 3, 3)	4.31%	10.11%

Table 7
Impact of full price discount on dynamic assortment.

Scenario	Price discount	Expected revenues		Revenue improvements
		Offer-all	Dynamic assortment	
1	0%	52.09	58.03	11.40%
2	10%	50.21	54.02	7.58%
3	20%	46.81	49.10	4.91%
4	30%	42.17	43.57	3.34%

increase after rationing inventory. However, under the offer-all policy, extra inventory may not necessarily result in revenue increase. In scenarios 2, 5 and 6, the extra inventory conversely decreases the expected revenues. Intuitively, we believe that more inventory will bring more revenues. But this belief is not true in our numerical experiments. It can be explained as follows. Products 2, 5 and 6 all have lower prices. But their utilities are not so low. If consumers choose these products in the offered assortment, the retailer's expected revenues will be hurt. We further can see that under both offer-all policy and dynamic assortment, extra inventory could produce much more expected revenue improvements for scenarios 3, 4, 7 and 8 than for other scenarios. These products have higher prices. So in order to increase revenues, the retailer can put extra inventory on products with higher prices. Finally it can be observed that extra inventory can produce more revenues under the dynamic assortment than under the offer-all policy.

5.4. Impact of initial price on the dynamic assortment performance

The prices of products significantly affect the retailer's revenues. Although we do not consider pricing decision in the context of dynamic assortment, we want to give some insights for the impact of initial prices on the performance of dynamic assortment. In supermarkets, it is so common that there are various price discounts on products. Table 7 reports the revenue improvements from dynamic assortment, under different price discounts. Here the price discounts apply to eight products of two brands simultaneously. Results in Table 7 show that offering price discount will decline the expected revenues under both offer-all policy and dynamic assortment. However, despite that, the dynamic assortment yields more revenues, in comparison with the offer-all policy. More importantly, increasing price discount will hurt the percentage of revenue improvements from the dynamic assortment. That is, price discount has a negative impact on the dynamic assortment. It is because price discount makes the difference among products' prices smaller and thus different products present similar utilities to consumers. Then the function of the dynamic assortment is weakened.

We finally examine the performance of dynamic assortment, when we only lower price of one product. Table 8 summarizes the results, where each scenario corresponds to reducing price of one product by 20%. Here the initial inventory is set equal to $\mathbf{y}_0 = \{1, 1, 2, 3, 1, 6, 3, 2\}$. "P1-20%" means that the price discount factor $\delta = 20\%$ only applies to P1. From Table 8, we can observe some meaningful insights. Under

Table 8
Impact of partial price discount on dynamic assortment.

Scenario	Price discount	Expected revenues		Revenue improvements
		Offer-all	Dynamic assortment	
1	P1–20%	51.08	57.98	13.53%
2	P2–20%	51.48	57.90	12.47%
3	P3–20%	51.00	56.61	11.01%
4	P4–20%	50.87	55.93	9.95%
5	P5–20%	51.85	58.00	11.87%
6	P6–20%	50.58	57.32	13.34%
7	P7–20%	50.71	56.25	10.92%
8	P8–20%	51.39	55.06	7.13%

dynamic assortment, having price discount on more expensive products within one brand leads to more revenue loss. Under offer-all policy, the same price discount on more expensive products within NB do not mean losing more revenues, e.g. cases 7 and 8. Although P7 and P8 are more expensive than P6, applying the same price discount to them brings more expected revenues than to P6. For products with similar prices, like P1 and P6, the same price discount yields different impacts on expected revenues under both policies. Expected revenues of P6%–20% decrease more than these of P1%–20%. Besides, for scenarios 1, 2, 5 and 6, we find that price discount on products whose prices are relatively lower within one brand would reinforce the performance of dynamic assortment.

6. Conclusions

We consider the dynamic assortment optimization problem, where a retailer has limited inventories of products of two brands from a category. The heterogeneous brands include one store brand and one national brand. We characterize consumer choice process using a two-stage NMNL model, in which a consumer first decides which brand or the no-purchase option to choose and then decides which product offered by that brand to purchase. In each period of the selling season, the retailer determines an assortment from a set of products offered to consumers given available inventory and the remaining time in the season so that the expected revenues are maximized.

Using the MCMC procedure based on the sales transaction data collected from a supermarket chain, we estimate consumer choice behavior. Our estimation results show that products of store brand and national brand are different in price sensitivity and consumer perceived utilities. And the products within one brand are closer substitution to each other than are products from another brand. Hence, it is necessary for the retailer to take brand heterogeneity into account when making operation decisions. We examine how the brand heterogeneity affects the performance of dynamic assortment. If the retailer is not able to distinguish between store brand and national brand while characterizing consumer choice behavior, the expected revenues will be significantly overestimated. And the degree of this revenue deviation will be affected by products' initial inventory level and prices.

We further examine the potential revenue improvements from implementing dynamic assortment by benchmarking a myopic policy where all available products are shown to any arriving consumers. We empirically show that factors like initial inventory levels and initial prices will affect the performance of dynamic assortment. More inventories could bring out more revenues and intensify the performance of dynamic assortment. But there are no significant revenue improvements when the inventory levels exceed some threshold. We also find that lowering products' prices not only reduces the expected revenues, but also weakens the performance of dynamic assortment. The same price discount on low-price products in one brand would reinforce the performance of dynamic assortment.

Our analytical results and case study suggest that it is necessary and valuable to distinguish store brand from national brand, and dynamic

Table 9
Estimation results for NMNL model.

Parameter		Store brand: JiaHui				National brand: YiBao			
		P1	P2	P3	P4	P5	P6	P7	P8
$\hat{\beta}_0$	Mean	3.2062	−1.8639	−0.9194	2.8945	−0.2783	0.8822	−1.3001	1.3577
	SD	0.2510	0.1011	0.2673	0.1759	0.2386	0.2017	0.1392	0.1019
	95% CI	[3.2013, 3.2111]	[−1.8659, −1.8619]	[−0.9247, −0.9142]	[2.8910, 2.8979]	[−0.2830, −0.2736]	[0.8783, 0.8862]	[−1.3029, −1.2974]	[1.3557, 1.3597]
$\hat{\beta}_1$	Mean	−5.1192	−1.6170	−1.9444	−1.8845	−1.1757	−0.0143	−0.0170	−0.5888
	SD	0.1942	0.0950	0.0253	0.0316	0.1870	0.0149	0.0189	0.0256
	95% CI	[−5.1230, −5.1153]	[−1.6189, −1.6152]	[−1.9449, −1.9439]	[−1.8851, −1.8838]	[−1.1793, −1.1720]	[−0.0146, −0.0140]	[−0.0173, −0.0166]	[−0.5893, −0.5883]
$\hat{\beta}_2$	Mean	0.7727	2.6304	2.5332	1.4797	0.8334	0.6168	1.0774	0.7229
	SD	0.2808	0.2031	0.0942	0.0270	0.2818	0.2637	0.1667	0.0155
	95% CI	[0.7672, 0.7782]	[2.6264, 2.6344]	[2.5313, 2.5350]	[1.4793, 1.4803]	[0.8278, 0.8389]	[0.6116, 0.6220]	[1.0741, 1.0807]	[0.7226, 0.7232]
$\hat{\mu}$	Mean	4.6069							
	SD	0.2062							
	95% CI	[4.6029, 4.6110]							
$\hat{\lambda}$	Mean	0.0348							
	SD	0.0008							
	95% CI	[0.0348, 0.0349]							

Table 10
Estimation results for MNL model.

Parameter		Store brand: JiaHui				National brand: YiBao			
		P1	P2	P3	P4	P5	P6	P7	P8
$\hat{\beta}_0$	Mean	5.9386	2.5232	−2.0867	−6.1612	4.3157	3.4323	−1.3021	−0.5705
	SD	0.1868	0.1936	0.1436	0.1018	0.0607	0.1839	0.1195	0.2341
	95% CI	[5.9350, 5.9423]	[2.5194, 2.5270]	[−2.0895, −2.0838]	[−6.1632, −6.1592]	[4.3145, 4.3169]	[3.4287, 3.4359]	[−1.3045, −1.2998]	[−0.5751, −0.5659]
$\hat{\beta}_1$	Mean	−3.9784	−1.1140	−1.0764	−3.7434	−1.9705	−0.0124	−0.0292	−0.3197
	SD	0.0765	0.0841	0.0509	0.0320	0.0449	0.0189	0.0206	0.0426
	95% CI	[−3.9799, −3.9769]	[−1.1156, −1.1123]	[−1.0774, −1.0754]	[−3.7441, −3.7428]	[−1.9714, −1.9696]	[−0.0128, −0.0120]	[−0.0296, −0.0288]	[−0.3205, −0.3188]
$\hat{\beta}_2$	Mean	4.7531	3.2542	2.8806	6.9571	0.0357	1.9471	3.2233	1.4075
	SD	0.2896	0.2768	0.0646	0.0188	0.1207	0.2473	0.1222	0.0230
	95% CI	[4.7474, 4.7588]	[3.2487, 3.2596]	[2.8793, 2.8819]	[6.9567, 6.9575]	[0.0334, 0.0381]	[1.9423, 1.9520]	[3.2210, 3.2257]	[1.4071, 1.4080]
$\hat{\lambda}$	Mean	0.0257							
	SD	0.0003							
	95% CI	[0.0257, 0.0258]							

assortment with heterogeneous brands is an effective lever for increasing retailers' revenues. Our empirical study can offer some advice on how to characterize consumer choice behavior and implement the dynamic assortment in the retail market.

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Appendix. Detailed estimation results

In this section, we detail the estimation results of parameters under different consumer choice models. The confidence intervals of the parameters are reported in Tables 9 and 10. In the MCMC estimation procedure, we discarded the first 90,000 estimation results and used the average of the last 10,000 sampled values as the estimand parameters. Besides we calculated the standard deviation of sampled parameters in the last 10,000 iterations, and then obtained the double-sided 95% confidence interval of each interval.

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